

Outlier Detection in Rating-Scale Data via Autoencoders

Max Welz^{1,2} Andreas Alfons¹

¹Erasmus University Rotterdam, Dept. of Econometrics

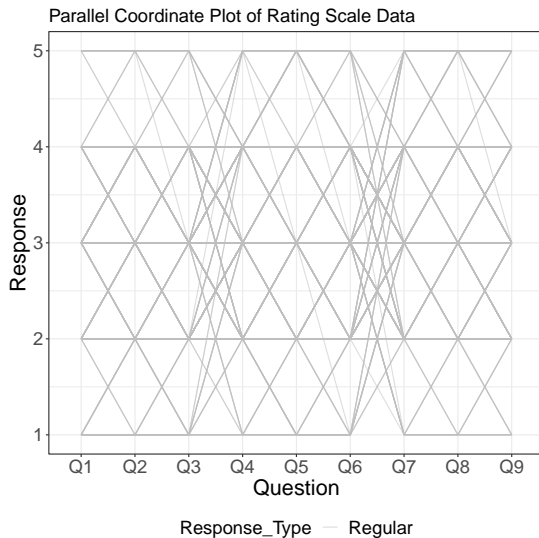
²Erasmus Medical Center, Dept. of Public Health

September 23, 2021

ICORS 2021, Vienna, Austria

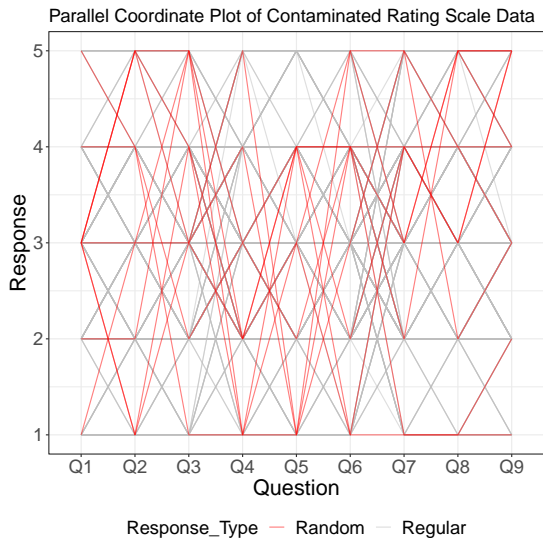


Parallel Coordinate Plot of Rating-Scale Dataset



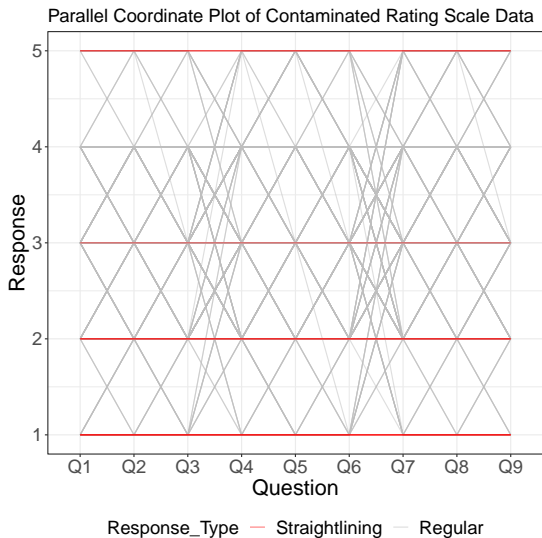
Erasmus

Parallel Coordinate Plot of Rating-Scale Dataset



Erasmus

Parallel Coordinate Plot of Rating-Scale Dataset



Erasmus

Types of Outliers in Rating-Scale Data

The psychological literature defines various types of rating-scale outliers. We focus on *content nonresponsivity* (Nichols et al., 1989):

- ① perfect straightlining;
- ② imperfect straightlining;
- ③ perfect extreme responding;
- ④ imperfect extreme responding;
- ⑤ random responding.



Type 1.1: Perfect Straightlining

Perfect straightlining is the tendency to consistently choose the same answer category, regardless of question content.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
ICORS 2021 is awesome.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
I like chocolate.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Oxygen is important.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
I like my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
I dislike my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Getting bitten by a shark would be fun.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>

Figure: Perfect straightlining with “Strongly Agree” as focal response.

Type 1.2: Imperfect Straightlining

Imperfect straightlining is the tendency to consistently choose responses around the same focal answer category, regardless of question content.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
ICORS 2021 is awesome.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
I like chocolate.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Oxygen is important.	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I like my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
I dislike my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Getting bitten by a shark would be fun.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>

Figure: Imperfect straightlining with “Agree” as focal response.

Type 2.1: Perfect Extreme Responding

Perfect extreme responding is the tendency to choose solely the most extreme answer categories, regardless of question content.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
ICORS 2021 is awesome.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
I like chocolate.	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Oxygen is important.	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I like my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
I dislike my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Getting bitten by a shark would be fun.	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Figure: Perfect extreme responding.

Erasmus

Type 2.2: Imperfect Extreme Responding

Imperfect extreme responding is the tendency to choose extreme answer categories (albeit not necessarily the most extreme category), regardless of question content.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
ICORS 2021 is awesome.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
I like chocolate.	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Oxygen is important.	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I like my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
I dislike my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Getting bitten by a shark would be fun.	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Figure: Imperfect extreme responding.

Erafus

Type 3: Random Responding

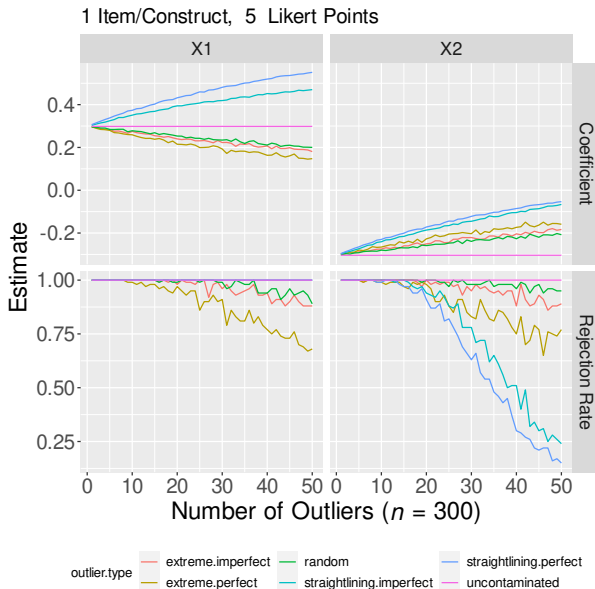
Random responding is the tendency to randomly choose answer categories, regardless of question content.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
ICORS 2021 is awesome.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
I like chocolate.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Oxygen is important.	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I like my job.	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I dislike my job.	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Getting bitten by a shark would be fun.	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Figure: Random responding.

Erafus

Effects of Outliers in Rating-Scale Data



Summary of Proposed Method

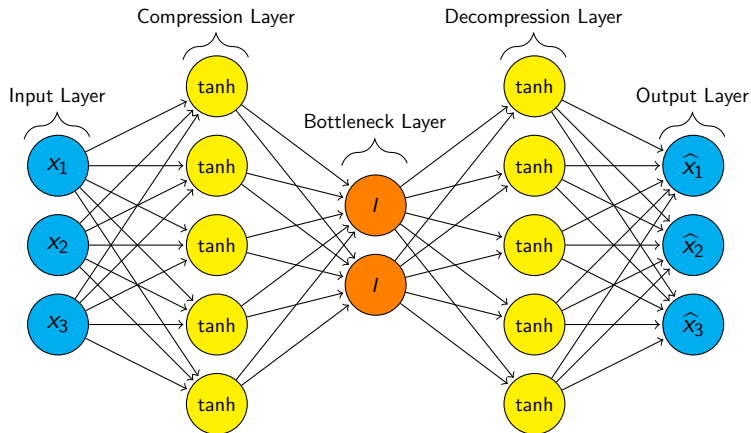
For outlier detection in rating-scale data, we propose to:

- Use a form of auto-associative neural networks (*autoencoders*; [Kramer, 1992](#));
- Transform the resulting outlier scores to central normality ([Raymaekers and Rousseeuw, 2021](#)).



Proposed Method (1/3)

- An autoencoder (Kramer, 1992) is a neural network with three hidden layers that attempts to reconstruct its input.
- Central hidden layer is crucial: compresses the input.
- Can be seen as nonlinear generalization of PCA (Kramer, 1991).



Erafuz

Proposed Method (2/3)

Use the per-individual mean squared reconstruction error as **outlyingness score** (OS) for each individual:

$$\text{OS}(\mathbf{x}) = \frac{1}{p} \sum_{j=1}^p \left(\frac{x_j - \hat{x}_j}{L_j} \right)^2,$$

where p is the number of questions, \hat{x}_j is the autoencoder's reconstruction of question response x_j , and L_j is the number of answer categories of question j .



Proposed Method (3/3)

We use the transformation by [Raymaekers and Rousseeuw \(2021\)](#).

- Let $\hat{g}(\cdot)$ be the rectified Box-Cox transformation.¹
- For scores OS_1, \dots, OS_n , robustly standardize the transformations by

$$z_i = \frac{\hat{g}(OS_i) - \text{median}\{\hat{g}(OS_j) : j = 1, \dots, n\}}{\text{MADN}\{\hat{g}(OS_j) : j = 1, \dots, n\}}.$$

- Flag observation i as *outlier* if

$$z_i > \sqrt{\chi_{0.975,1}^2}.$$

- Only allows for flagging in the right tail. Flagging in the left tail is work in progress (more on this later).

¹We currently experiment with various choices of the rectification constant C_ℓ to find a recommended choice. Currently $C_\ell = 0.25$.

Simulation Design (1/2)

Data generating process:

- We generate $n = 300$ correlated rating-scale observations by using the sampling scheme in [Kaiser et al. \(2011\)](#);
- We consider three constructs, each consists of five questions (i.e. $p = 3 \times 5 = 15$ questions);
- Questions *within* the same construct have high correlation of 0.8;
- Questions from different constructs have medium correlation of ± 0.3 ;
- Each question has five answer categories;
- Each dataset is contaminated with up to 50 outliers;
- We average the considered performance measures over $R = 100$ such datasets.



Simulation Design (2/2)

Benchmark methods:

- *Local Outlier Factor* (LOF; Breunig et al., 2000);
- G_+ score (from psychology; Zijlstra et al., 2007);
- Robust Mahalanobis distance via MCD (Rousseeuw, 1984);
- ... and seven more, as in Zijlstra et al. (2011).

Performance measures:

- % True Positives = fraction of true positives. “How many of the true outliers are flagged?”
- % False Positives = fraction of false positives. “How many of the flagged points are no outliers?”

Erafus

Simulation Results (1/5)

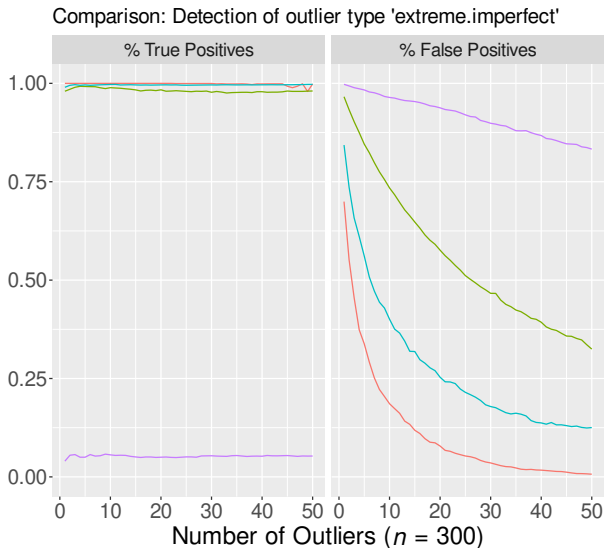
Comparison: Detection of outlier type 'random'



method — autoencoder — Gplus — LOF — MCD

Ezraus

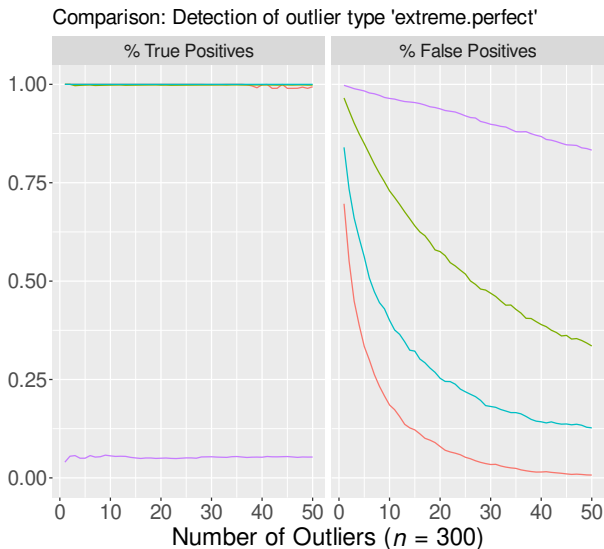
Simulation Results (2/5)



method — autoencoder — Gplus — LOF — MCD

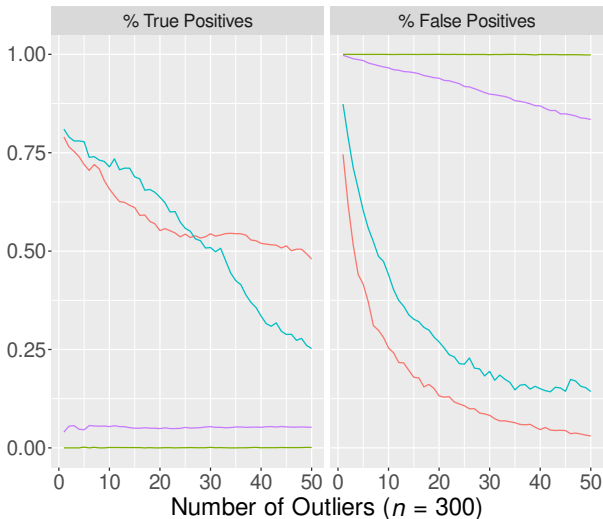
Ezra

Simulation Results (3/5)



Simulation Results (4/5)

Comparison: Detection of outlier type 'straightlining.imperfect'

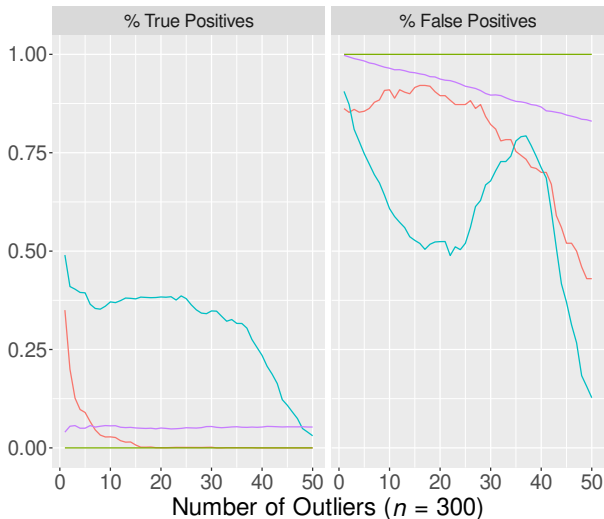


method — autoencoder — Gplus — LOF — MCD

Ezra

Simulation Results (5/5)

Comparison: Detection of outlier type 'straightlining.perfect'



method — autoencoder — Gplus — LOF — MCD

Ezra

Simulation Conclusion

- Our method outperforms all benchmark methods and reliably detects all types of rating-scale outliers, except perfect straightliners;
- Perfect straightliners should be easy to detect with an additional rule:
 - Either adapt autoencoder or outlyingness score;
- Gap between our method and the benchmark methods widens in more complex scenarios.



General Conclusion

- Rating-scale outliers are different from conventional outliers, but they can be just as harmful;
- Autoencoders seem to be promising in detecting rating-scale outliers;
- Robust methods for rating-scale data/categorical data are underdeveloped. **Potential for novel research ideas!**

Erasmus

**Thank you for the attention and a special
thanks to the organizers of ICORS 2021!
Let's have a discussion!**

(Slides: [https://mwelz.github.io/publications/.](https://mwelz.github.io/publications/))

A stylized, handwritten-style logo of the word "Erasmus" in a dark blue or black color.

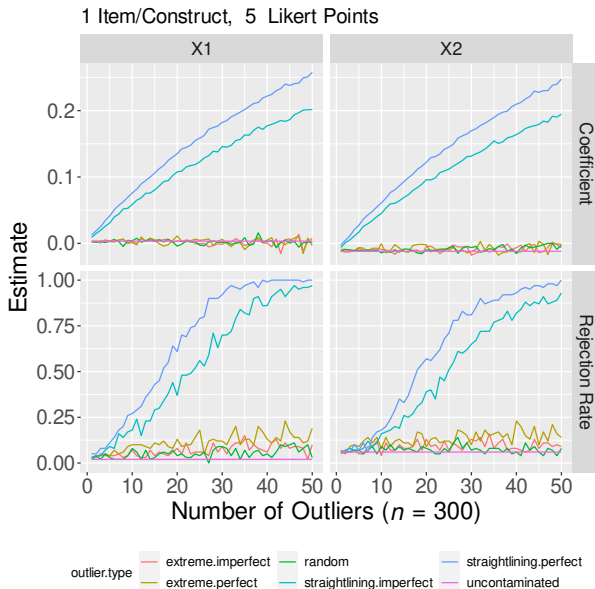
References

- Breunig, M. M., Kriegel, H.-P., Ng, R. T., and Sander, J. (2000). Lof: identifying density-based local outliers. In *Proceedings of the 2000 ACM SIGMOD international conference on management of data*, pages 93–104.
- Kaiser, S., Träger, D., and Leisch, F. (2011). Generating correlated ordinal random values. Technical Report 94, Department of Statistics, University of Munich.
- Kramer, M. A. (1991). Nonlinear principal component analysis using autoassociative neural networks. *AIChE Journal*, 37(2):233–243.
- Kramer, M. A. (1992). Autoassociative neural networks. *Computers & Chemical Engineering*, 16(4):313–328.
- Nichols, D. S., Greene, R. L., and Schmolck, P. (1989). Criteria for assessing inconsistent patterns of item endorsement on the MMPI: Rationale, development, and empirical trials. *Journal of Clinical Psychology*, 45(2):239–250.
- Raymaekers, J. and Rousseeuw, P. J. (2021). Transforming variables to central normality. *Machine Learning*. In press.
- Rousseeuw, P. J. (1984). Least median of squares regression. *Journal of the American Statistical Association*, 79(388):871–880.
- Zijlstra, W. P., van der Ark, L. A., and Sijtsma, K. (2007). Outlier detection in test and questionnaire data. *Multivariate Behavioral Research*, 42(3):531–555.
- Zijlstra, W. P., van der Ark, L. A., and Sijtsma, K. (2011). Outliers in questionnaire data: Can they be detected and should they be removed? *Journal of Educational and Behavioral Statistics*, 36(2):186–212.

Appendix

Erasmus

Effects of Outliers in Rating-Scale Data



Design Choices for the Autoencoder

- Use robust pseudo-Huber loss for fitting:

$$x \mapsto \delta^2 \left(\sqrt{1 + (x/\delta)^2} - 1 \right),$$

where $\delta > 0$ is a fixed constant.

- Central hidden layer: hyperbolic tangent activation (nonlinear):

$$x \mapsto \tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

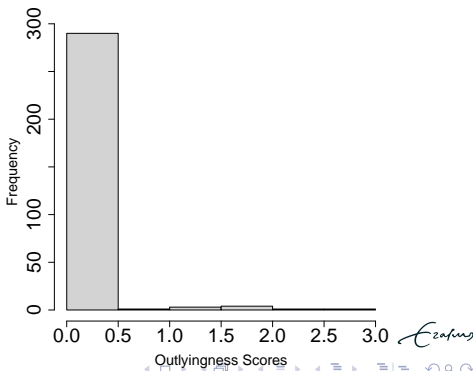
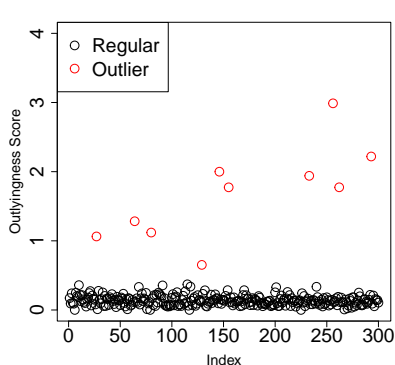
- Left and right hidden layers: linear *identity mapping*, $x \mapsto I(x) = x$; suggested by [Kramer \(1992\)](#).
- Use batch learning to avoid “too much” overfitting.

E. Zafra

Outlyingness Scores of Random Respondents

We apply our autoencoder on a simulated rating-scale dataset with $n = 300$ individuals, of which 10 are **random outliers**. Their outlyingness scores are clearly separated.

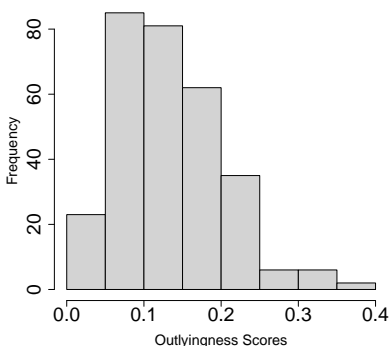
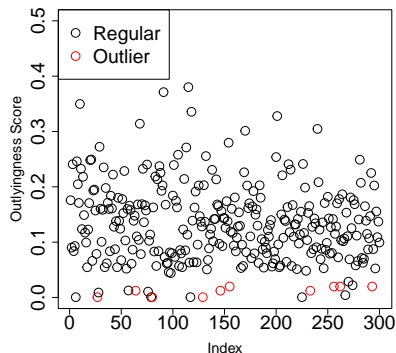
- Same situation for all other types of outliers EXCEPT perfect straightliners.



Outlyingness Scores of Perfect Straightliners

We repeat the previous exercise, but this time, the 10 outliers are **perfect straightliners**. Their outlyingness scores tend to be among the very lowest.

- Not unsurprising; straightliners are easy to reconstruct;
- Outliers can be in both tails of the scores' distribution!



Erafus